Effect of two-way coupling on the calculation of forest fire spread: model development


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Abstract. The present work addresses the problem of how wind should be taken into account in fire spread simulations. The study was based on the software system FireStation, which incorporates a surface fire spread model and a solver for the fluid flow (Navier–Stokes) equations. The standard procedure takes the wind field computed from a single simulation in the absence of fire, but this may not be the best option, especially for large fires. The two-way coupling method, however, considers the buoyancy effects caused by the fire heat release. Fire rate of spread is computed with the semi-empirical Rothermel model, which takes as input local terrain slope, fuels properties and wind speed and direction. Wind field is obtained by solving the mass, momentum and energy equations. Effects of turbulence on the mean flow field are taken into account with the $k–\varepsilon$ turbulence model. The calculation procedure consists of an interchange between the fire spread model and the wind model through a dynamic interaction. The present work describes the first part of this research, presenting the underlying models and a qualitative sensitivity analysis. It is shown that the update frequency for the dynamic interaction markedly influences the total calculation time. The best strategy for updating the wind field during the fire progression is presented. The dependence of results on mesh size is also described.

Additional keywords: fire spread simulation, fire–wind interaction, surface fire, wind simulation.

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Introduction

Prediction of forest fire spread has been a challenge to researchers and managers for several decades, but, in spite of the various models and fire behaviour prediction systems that have been developed as described by Sullivan (2009a, 2009b, 2009c), there is not yet a commonly accepted fire behaviour simulator that can be applied in operational conditions for large and complex fires. This is due to the complexity of the physical and chemical processes that are involved in large-scale fires requiring a large number of input parameters that are not easy to obtain and physical models that are not yet fully developed. Physical models have been formulated mathematically and implemented in numerical codes like Grishin (1994), Albini (1996), Linn et al. (2002), Séro-Guillaume and Margerit (2002) and Mell et al. (2007). Nonetheless, practical applications of these models are not yet possible owing to large computational requirements and uncertainty regarding the description of key physical processes. Evaluation of some of these models showed errors of up to an order of magnitude (e.g. Marino et al. 2012).

Simpler semi-empirical models to predict fire spread like the one developed by Rothermel (1972, 1983) have been of more effective use in fire behaviour prediction both in scientific and in operational work (Finney 1998; Andrews et al. 2008). In spite of its limitations, like the fairly small number of fire spread regimes that it considers and the limited range of conditions for which it was developed and validated, Rothermel’s model developed for surface fire spread prediction is still one of most widely applied in fire simulators. In the present study, we shall assume this scope of application as well, although other fire spread modes like crown fires and spot fires can be added to the fire simulator using adequate models to link surface to crown and spot fires.

It is commonly accepted that the fuel bed properties, terrain slope and wind velocity are the most important factors affecting fire behaviour, namely its rate of spread (ROS). Many studies have shown the particular importance of wind velocity near the fire front (see Clark et al. 1996a, 1996b; Viegas 2004a; Linn and Cunningham 2005) and the role of fire-induced wind in modifying fire spread (see Viegas 2004a, 2006). The present work will be dedicated mostly to this parameter and to its role on fire behaviour. Atmospheric wind flow is a boundary layer-type turbulent flow of air that is modified by the local topography, the level of instability of the atmosphere and its interaction with roughness elements like vegetation, houses, rocks and other ground-cover obstacles upwind from the fire front. In order to estimate fire behaviour correctly, it is necessary to predict changes in wind velocity components induced by the presence of the fire in the portion of the atmospheric boundary layer.
surrounding the area of the fire. A forest fire affects local wind conditions considerably. The wind patterns that would exist in the absence of a fire are replaced by localised flow structures resulting from the interaction between the atmospheric wind and the convection induced by the flaming combustion front. Several authors have analysed the coupling between the fire and the atmospheric flow (Clark et al. 2004; Viegas 2004a, 2006; Coen et al. 2013) but this has not yet been applied in fire simulators that can be used operationally.

Models for simulation of wind over complex topography may be classified into two main categories: prognostic and diagnostic models. Prognostic models operate at large scales, with mesh sizes typically larger than 2–4 km. They describe the dynamics of the atmospheric boundary layer, including predictive capabilities for the variables field, according to the diurnal cycle. Weather Research and Forecasting (WRF) (Coen et al. 2013) and (Mesoscale Model (MEMO) (Flissak and Moussiopoulos 1988) fall into this category. As pointed out in Forthofer et al. (2014), owing to the poor mesh resolution, prognostic models are not able to capture flow effects from topography with significant interest for fire simulation. Nevertheless, operating with a large simulation domain, prognostic models constitute a good source of input data for finer-mesh simulations. Diagnostic models provide a steady-state simulation and may be divided into two types. Mass-consistent models (also known as kinematic models) take as input a first guess of the velocity field obtained from interpolation of the input data, and modify it so as to achieve a divergence-free solution. The stability of the atmosphere and other thermal effects are simulated by adjusting the horizontal and vertical velocity components. The Nuatmos (Ross et al. 1988) and WindNinja (Forthofer et al. 2009) models fall into this category. Kinematic models require very little computational effort. As shown in Lopes (2003) and in Forthofer et al. (2014), they provide rather good solutions upwind and above terrain features, but often fail on the lee side, especially if fire separation is present. Navier–Stokes solvers, such as WindWizard (Forthofer et al. 2014) and WindStation (Lopes 2003), representing the second type of diagnostic model, constitute a more realistic approach, solving for mass and momentum (Navier–Stokes equations) conservation, and often include some form of turbulence closure as well. Application of such models to atmospheric flows has been successfully done in the past, with the Askervein experiment representing one of best-known benchmark cases (Raithby and Stubley 1987; Castro et al. 2003; Lopes 2003; Forthofer et al. 2014).

Several authors have coupled fire spread algorithms with weather models (e.g. Clark et al. 2004; Coen et al. 2013). The WRF-Fire model (Coen et al. 2013) is based on the WRF software, which is a prognostic weather model developed for large-spatial-scale simulations. The WRF-Fire version includes a model for coupling fire and wind. Coen et al. (2013) provide a rather detailed description of this model, with application to a test case on flat terrain. WRF is configured in LES (Large Eddy Simulation) mode, in order to be applicable to smaller scales (~40-m mesh spacing). On the simulations reported in the latter work, zero constant slope should be considered, in order to use periodic boundary conditions. It is not clear how the model would perform in a real complex terrain situation with a real fire situation and what the limits are as concerns spatial resolution and associated numerical errors and computational effort. The WRF model has also been used for providing initial conditions and boundary conditions for the CAWFE (Coupled Atmosphere–Wildland Fire Environment) prediction model (Coen 2013). This model was applied to a specific wildland fire (Coen and Riggan 2014) and, according to the authors, the main distinguishing fire features were correctly depicted by the simulation. Numerical weather prediction models, such as WRF, may also be used for improving the accuracy of simpler kinematic models through a downscaling process, as demonstrated in the work by Wagenbrenner et al. (2016).

The present study describes an evolution of the fire simulator (FireStation) presented in Lopes et al. (2002), now incorporating a wind calculation module that takes into account feedback from the convective flow induced by the fire on the atmospheric flow and its effect on the spread of the fire. This wind calculation engine is based on the original WindStation model (Lopes 2003), including additional features such as, among others, the inclusion of the energy equation and more flexibility in the assignment of incident velocity profiles. The present paper describes the underlying physical and numerical models, as well as simplified simulation scenarios. The fundamental equations for fluid flow and turbulence modelling are presented, along with a brief description of the numerical methodology for their solution. Next, models for fire spread are described, including fire ROS, fire growth and interaction between wind and fire. A qualitative sensitivity analysis of this methodology is undertaken, where the influence of the update frequency for the dynamic interaction between the fire and the wind is studied. The influence of mesh resolution for wind calculation is also analysed.

### Theory background

#### Wind field modelling

**Governing equations**

In the present approach, the Boussinesq approximation is adopted, considering that density is constant except in its contribution to buoyancy forces. As the rate of change in boundary conditions due to fire is very low, the flow is solved using a steady-state approach. In fact, air flow velocity is much greater than fire spread velocity. This means that, during the time needed for a fluid parcel to travel through the fire-affected area, the heat release rate distribution at the ground due to fire remains essentially unchanged – thus, the fluid flow may be regarded as behaving in a steady-state manner with respect to fire.

The Navier–Stokes equations represent momentum conservation for a fluid flow. Considering the simplifications just described, the formulation in a Cartesian coordinate system (see e.g. White 2011, p. 247), for a steady-state condition and constant density, is:

\[
\frac{\partial}{\partial x_i} (\rho u_i u_j) = \frac{\partial}{\partial x_j} \left[ \rho \frac{\partial u_i}{\partial x_j} \right] + \frac{\partial}{\partial x_i} \left[ \Gamma \frac{\partial u_i}{\partial x_j} \right] - \frac{\partial p}{\partial x_j} + B_i
\]

where \( \rho \) = \((u, v, w) \), \( x_i = (x, y, z) \) for \( i = (1, 2, 3) \), \( \rho \) is the fluid density, \( p \) is pressure, and \( \Gamma = \rho \mu_p \) + \( \mu \) is the diffusion coefficient for momentum, i.e. the effective viscosity, which is a
function of the turbulence variables, as described below. Buoyancy forces are present only for the vertical momentum component ($i = 3$) and are given by:

$$ B_3 = \rho_{ref} \gamma g \beta (T - \theta) $$

(2)

where $\beta$ is the thermal expansion coefficient, $g$ is gravity, $T$ is the temperature and $\theta$ is the temperature corresponding to the dry adiabatic lapse rate ($-10^\circ C km^{-1}$). Thus, in the absence of other thermal effects, the neutral atmosphere will produce no buoyancy forces.

Mass conservation is described by the continuity equation, for which the steady-state formulation is:

$$ \frac{\partial}{\partial x_i} (\rho u_i) = 0 $$

(3)

The temperature field, necessary for the computation of buoyancy forces, is obtained through the solution of the energy equation:

$$ \frac{\partial}{\partial x_i} (\rho c_p u_i T) = \frac{\partial}{\partial x_i} \left( T \frac{\partial T}{\partial x_i} \right) + S_T $$

(4)

In this case, the diffusion coefficient $\Gamma$ is obtained as:

$$ \Gamma = \left( \frac{\mu}{P_r} + \frac{\mu_s}{P_r s} \right) c_p $$

(5)

with $c_p$ as the air specific heat and $P_r$ and $P_{rs}$ as the laminar and the turbulent Prandtl numbers respectively. The source term $S_T$ in Eqn 4 accounts for energy exchanged with the ground, which includes heat exchange by the fire. Although not considered in the present work, this term may include modelling other heat sources through raster mapping to take into account, for example, solar radiation heating. Alternatively, a specified temperature field may be imposed at the ground.

As turbulence effects on fluid flow may occur on a very large range of time and length scales, obtaining the solution of the transport equations for the instantaneous variables field is not feasible for most practical applications, including atmospheric flows, where length scales may range from centimetres up to kilometres. As a work-around, eddy viscosity turbulence models simulate the effects of turbulence on the mean flow field by increasing the effective viscosity. One of the most widely used is the $k - \varepsilon$ turbulence model, proposed by Launder and Spalding (1972, 1974). The turbulence kinetic energy ($k$), which is a measure of the flow velocity fluctuations, is related to turbulence intensity $T_i$ as:

$$ T_i = \frac{\sqrt{2} k}{V} \Rightarrow k = \frac{3}{2} (T_i V^2) $$

(6)

where $V$ is the local average velocity magnitude. The dissipation rate of turbulence kinetic energy ($\varepsilon$) is related to the dissipation length scale $L$ as follows:

$$ L = C_{\mu} \frac{k^{3/2}}{\varepsilon} \Rightarrow \varepsilon = C_{\mu} \frac{k^{3/2}}{L} $$

(7)

Turbulence kinetic energy and its dissipation rate are used to compute turbulence viscosity with the following equation:

$$ \mu_t = C_{\mu} \frac{\rho k^2}{\varepsilon} $$

(8)

In the original formulation of the $k - \varepsilon$ model, the ‘constant’ $C_{\mu}$ was adjusted to the value $C_{\mu} = 0.09$, based on laboratory experiments for flow over a smooth plate. As stated in Raithby and Stubble (1987), this value is to be corrected for the case of atmospheric flows. Data analysis from several atmospheric studies by Panofsky and Dutton (1984) provided $C_{\mu} = 0.033$ for a wide range of surface conditions and scales. This is the value adopted in the present work. The turbulence kinetic energy and its dissipation rate are described by the following transport equations:

$$ \frac{\partial}{\partial x_j} (\rho u_j k) = \frac{\partial}{\partial x_j} \left( \mu + \frac{\mu_s}{\sigma_t} \frac{\partial k}{\partial x_j} \right) + P_t + G - \rho \varepsilon $$

(9)

$$ \frac{\partial}{\partial x_j} (\rho u_j \varepsilon) = \frac{\partial}{\partial x_j} \left( \left( \mu + \frac{\mu_s}{\sigma_t} \right) \frac{\partial \varepsilon}{\partial x_j} \right) + \frac{\varepsilon}{k} (C_1 P_t + C_1 C_3 G - C_2 \rho \varepsilon) $$

(10)

where $\sigma_t$ and $\sigma_e$ are Prandtl numbers. The term $P_t$ represents the production rate of $k$ as the result of the velocity gradients:

$$ P_t = -\rho u_j \frac{\partial u_j}{\partial x_j} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \right) \frac{\partial u_i}{\partial x_j} $$

(11)

The term $G$ (Markatos et al. 1982; Henkes et al. 1991) represents the production or destruction of turbulence due to buoyancy forces and has the following formulation:

$$ G = -\beta g \frac{\mu_s}{\sigma_t} \frac{\partial T}{\partial z} $$

(12)

The model constants are:

$$ \sigma_t = 1.0, \sigma_e = 1.3, C_1 = 1.45, C_2 = 1.9, C_3 = \tanh \frac{w}{\sqrt{u'^2 + v'^2}} $$

(13)

Effects of buoyancy on turbulence dissipation are less understood than the effects on turbulence kinetic energy and there is no consensual formulation for this. The term $C_3$, as formulated above (Henkes et al. 1991), cuts buoyancy effects on dissipation for horizontal shear layers and progressively allows their influence as the velocity vector approaches the vertical direction. The remaining constants are given in Launder and Spalding (1972, 1974).

**Boundary conditions and initialisation**

At the ground boundary, a logarithmic law is assumed for velocity variation with height, $z$, above the ground:

$$ \frac{V}{V_s} = \frac{1}{k} \ln \left( \frac{z}{z_0} \right) $$

(14)
where $V$ is the velocity magnitude, $\kappa = 0.41$ is the von Karman constant and $z_0$ is known as the roughness parameter, which is taken as a fraction $f$ of a typical roughness height ($h_0$) through the equation $z_0 = fh_0$. Values for $f$ range from 0.03 to 0.25, depending on the layout and shape of the roughness elements (ground cover elements such as vegetation). A conservative value of $f = 0.15$ is recommended by Plate (1982) for most natural surfaces. The ratio $z/z_0$ must always be higher than unity, which means that the height of the first calculation level (centre of the control volume adjacent to the ground) should be higher than the roughness parameter $z_0$. The friction velocity $V^*$ is a measure of the ground shear stress, $\tau_w = \rho V^2$ and provides the boundary condition for the momentum equations:

$$\tau_w = \rho \left( V \kappa / \ln \left( \frac{z}{z_0} \right) \right)^2$$  \hspace{1cm} (15)

Eqn 15 is evaluated at the first computational node near the ground.

Boundary conditions for the turbulence quantities are obtained based on the hypothesis of local equilibrium. It can be proved that, based on this assumption, turbulence kinetic energy is given by (Raithby and Stubble 1987; Castro et al. 2003):

$$k = \frac{V^2}{\sqrt{C_\mu}}$$  \hspace{1cm} (16)

which constitutes the boundary condition for $k$. The dissipation rate near the ground is given by:

$$\varepsilon = \frac{V^3}{kd}$$  \hspace{1cm} (17)

with $d$ as the distance of the first node to the ground.

For the energy equation, the ground boundary condition is the heat flux rate exchange with the ground. If a local fire exists, the propagation model provides this value, to be included in the term $S_T$, in Eqn 4. This is computed with Eqn 30, to be presented below. If no fire exists, an adiabatic boundary is a reasonable assumption.

At downstream and lateral boundaries, a null gradient condition is imposed for all variables (zero first derivative). One should be aware that boundaries are always subject to some uncertainty; consequently, the wind calculation domain should always be larger than the area of interest, in order to minimise the effect of these boundaries on the region of interest.

For the top boundary, null gradients are assigned to all variables.

Initial values must be assigned to the whole 3-D wind calculation domain – this includes velocity magnitude and direction, as well as turbulence quantities and temperature. Boundaries where inflow is verified are considered as inlet boundaries, with conditions that remain unchanged during the calculation. Temperature values are obtained from a specified profile or computed from a selected class of atmospheric stability. For velocity and turbulence quantities, fully developed profiles typical of atmospheric flows are internally available in the software, such as the example shown in Fig. 1. Velocity field initialisation may also be done based on data available from meteorological stations. In this case, wind speed and direction at each meteorological station are employed to compute the shape of the local boundary layer, using a power law equation of the following form:

$$V/V_\infty = \left( \frac{z}{\delta} \right)^{1/a}$$  \hspace{1cm} (18)

where $V_\infty$ is the velocity at the top of the boundary layer, $\delta$ is the boundary layer height and $z$ is an exponent characterising the boundary layer shape (Plate 1971). The profile, characterised by the values $V_\infty$, $\delta$ and $a$, is determined from knowledge of the velocity at two heights, and an estimated value of the boundary
layer height. It is assumed that wind direction is constant with height at the station location. Values at each computational mesh node are calculated through bi-linear interpolation from each available station location, using the inverse of the distance as the weighting factor.

**Numerical solution**

The mesh where the flow solution takes place is of a structured type (regular arrangement of parallel epipedic elements), corresponding to a rectangular computational domain. The mesh is Cartesian with uniform spacing in the horizontal (\(x, y\)) plane, running from west to east and from south to north, as exemplified in Fig. 2a. In order to properly describe the topographic characteristics, a terrain-following (boundary-fitted) coordinate system is adopted in the vertical direction, as depicted in Fig. 2b. The mesh is generated algebraically, using a gradual variable expansion factor in the vertical direction, computed so as to satisfy the specified number of vertical levels, the near-ground first-level height and total domain height, ensuring a smooth transition in the mesh spacing.

The transport equations are transformed from their original Cartesian form into a generalised coordinate form using the chain rule:

\[
\frac{\partial \phi}{\partial x_i} = \frac{\partial x_i}{\partial \xi} \frac{\partial \phi}{\partial \xi} + \frac{\partial x_i}{\partial \eta} \frac{\partial \phi}{\partial \eta} + \frac{\partial x_i}{\partial \zeta} \frac{\partial \phi}{\partial \zeta} = \xi_u \frac{\partial \phi}{\partial \xi} + \eta_u \frac{\partial \phi}{\partial \eta} + \zeta_u \frac{\partial \phi}{\partial \zeta}
\]  

(19)

where terms like \(\xi_u\) are the contravariant metrics of the transformation, relating the computational coordinates (\(\xi, \eta, \zeta\)) and the physical coordinates \(x_i (x_1 = x, x_2 = y, x_3 = z)\), and \(\phi\) is a generic variable (\(u, v, w, k, e\) or \(T\)). After some mathematical manipulation, the transport equation for a generic variable \(\phi\) is (see Lopes et al. 1995 for additional information):

\[
\frac{\partial}{\partial \xi_j} (J \rho U_j \phi) = \frac{\partial}{\partial \xi_j} \left[ J \Gamma \left( g \gamma \frac{\partial \phi}{\partial \xi_j} \right) \right] + b
\]  

(20)

where \(U_j\) is a generic contravariant velocity, given by:

\[
U_1 = \xi_u u + \xi_v v + \xi_w w; \quad U_2 = \eta_u u + \eta_v v + \eta_w w; \quad U_3 = \zeta_u u + \zeta_v v + \zeta_w w
\]  

(21)

and \(J\) is the Jacobian of the transformation:

\[
J = \begin{vmatrix}
    x_\xi & x_\eta & x_\zeta \\
    y_\xi & y_\eta & y_\zeta \\
    z_\xi & z_\eta & z_\zeta \\
\end{vmatrix}
\]  

(22)

where the quantities in the determinant are the co-variant metrics, which are a function of the contravariant metrics. In turn, \(b\) includes all terms not fitting in the diffusion term (cross derivatives, momentum sources due to temperature, pressure gradients and other terms, depending on the particular concretisation for \(\phi\)). The generalised coordinate form of the mass conservation equation is:

\[
\frac{\partial}{\partial \xi_j} (J \rho U_j) = 0
\]  

(23)

Owing to fact that the mesh is Cartesian in the \((x, y)\) plane and the vertical mesh lines are straight, some of the metrics are null (e.g. \(\xi_y, \zeta_x, \chi_z\)). This allows some simplifications in the final form of the generalised equations, with benefits in terms of computational time and memory requirements. After discretisation and integration, the equations are cast in the general algebraic form:

\[
a_P \phi_P = \sum_{ab} a_{ab} \phi_{ab} + b
\]  

(24)

This equation relates the value of the generic variable \(\phi\) (velocity components, turbulence quantities and temperature) at location \(P\) with its neighbour \((nb)\) values. Coefficients \(a_P\) and \(a_{ab}\) account for diffusion and advection fluxes.
A control volume approach is adopted for the integration of the transport equations. The SIMPLEC algorithm (Van Doormaal and Raithby 1984), which is a modification of the original SIMPLE algorithm proposed by Patankar (1980), is employed as a means for the segregated solution of the primitive Cartesian velocity components \((u, v, w)\) and pressure. To account for the non-staggered mesh arrangement, where all variables are located at the control volume (CV) centres, the Rhie–Chow interpolation procedure (Rhie and Chow 1983) is implemented.

The iterative process is considered to be converged when all the normalised residuals are lower than a predefined value \(R_{\text{conv}}\):

\[
\text{Max}(R_u, R_v, R_w, R_m, R_k, R_c, R_f) < R_{\text{conv}}
\]

(25)

The total normalised residual for the transport equations is determined as follows:

\[
R_\phi = \frac{\left| \sum_{\phi} \left( \frac{\alpha_p \phi_P - \sum_{\phi_b} \phi_{\phi_b} + h_{\phi_P}}{\alpha_p (\phi_{\text{max}} - \phi_{\text{min}})} \right) \right|}{\sum_v}
\]

(26)

where \((\phi_{\text{max}} - \phi_{\text{min}})\) quantifies the amplitude of the variable \(\phi\) in the calculation domain and \(v\) is the volume of each computational cell.

**Fire spread modelling**

**Fire rate of spread**

The fire behaviour model implemented in the present work is based on Rothermel’s surface fire spread model (Rothermel 1972), mainly owing to the latter’s well-established character and versatility of application. This is a semi-empirical model developed to be used under a wide range of conditions. Rothermel’s model takes as input fuel characteristics, wind speed and direction and terrain slope, and provides as output the fire ROS along the main spread direction. The fundamental equation of this model expresses an energy balance within a unit volume of the fuel ahead of the flame. It illustrates the concept that the ROS is governed by the ratio of fuel heating and subsequent ignition. The symbol \(h_r\) represents the heat sink, with \(\rho_b\) as the bulk density, which is the mass of fuel per unit volume, \(\tilde{e}\) is the effective heating number, quantifying the ratio between the bulk density and the mass of fuel involved in the ignition process, and \(Q_{\text{b}}\), the so-called heat of pre-ignition, is the heat required to bring a unit mass of fuel to ignition including the energy to dehydrate the fuel. The basic rate of spread \(R_b\) is affected by the presence of wind and terrain slope through the factors \(\tilde{e}_{\text{wind}}\) and \(\tilde{e}_{\text{slope}}\), respectively. Both these factors act directionally as a conditioning factor on the ROS. As may be seen, each contribution (wind and slope) has its own direction, which is added to the omnidirectional contribution of the zero-slope, zero-wind, basic ROS. Wind velocity for application of the previous equation should be taken at a vertical distance from the ground dependent on the midflame height.

Albini and Baughman (1979) considered the average value resulting from the integration of the logarithmic law between the vegetation top and the flame height as follows:

\[
\frac{V_f}{V_1} = \frac{1 + 0.36(h_f/h_t)}{\ln \left( \frac{h_f}{h_t} \right)} - 1
\]

(28)

where \(V_f\) is the computed average wind velocity value, \(V_1\) is the wind velocity computed at the first vertical level, \(h_f\) stands for flame height and \(h_t\) is the vegetation height. All quantities in Eqn 27 are computed using fuel characteristics as defined by the fuel model concept (Albini 1976; Anderson 1982). These characteristics include fuel load, fuel particle heat content, surface-to-volume ratio or fuel bed depth. The environment characteristics are also taken into account with parameters related to fuel moisture. Most of fuel properties are supplied for different fuel size classes and dead–live classification. Because different size classes are allowed, fuel load,
surface-to-volume ratio and moisture content are averaged using the total surface area as the weighting factor, following the suggestion of Rothermel (1972). Dead and live components are treated separately, each being used to compute its own reaction intensity. The final value for the reaction intensity is obtained as the sum of the corresponding values for dead and live fuels.

Apart from fire ROS, Rothermel’s model provides several output quantities concerning fire behaviour, such as fireline intensity (W m\(^{-1}\)), flame length (m), residence time (s) and rate of heat release, also referred as reaction intensity \(I_r\) (W m\(^{-2}\)). This last variable describes the fire energy source for the wind–fire interaction calculation. Residence time multiplied by the fire ROS allows determination of the flame front depth (m), which, along with the fire perimeter, defines the burning region. When a flame extinguishes, which occurs after the local burning time exceeds the residence time, an exponential decay is assigned to \(I_r\), corresponding to the smouldering phase. This is computed according to the following equation:

\[
I_{rd}(t) = f_d I_r e^{-t/t_d}
\]

where \(t\) represents time, \(I_{rd}\) is the reaction intensity during the decay phase, \(f_d\) is an initial reaction intensity cut-off due to flame extinguishment and \(t_d\) is a relaxation time characteristic of the fuel.

**Fire shape**

Although Rothermel’s model describes fire ROS along the maximum spread direction, it does not provide information

\[\text{Wind speed (m s}^{-1}\)\]

\[\text{Temp. (°C)}\]

*Fig. 5. Velocity field 10 m above ground and burned area shape. (a) No fire situation; (b) 450 ha of burned area; (c) 1000 ha burned area; (d) vertical plane visualisation along the fire central line depicted in (c). Domain height is 2000 m.*
about ROS along other directions. This information is, nevertheless, necessary to compute fire shape. Several models exist for fire shape, mostly based on ellipses whose geometrical parameters are given as a function of the wind speed (e.g. Green et al. 1983). The effect of topography is modelled through the addition of a slope-equivalent wind speed that would produce the same effect as the actual slope on the fire ROS. The ‘effective’ wind speed, to be considered on the fire shape model, is obtained through the vectorial summation of the actual wind velocity and the equivalent wind velocity due to slope.

The model proposed by Anderson (1983), describing fire shape through a double ellipse, as depicted in Fig. 3a, represents, according to the authors, the best practical realistic approximation for observed fire shapes under a variety of circumstances. This model, nevertheless, does not cope with zero-wind situations. In the present implementation, when the local equivalent wind speed at midflame height falls below 0.2 m s$^{-1}$, the double ellipse formulation is replaced by a single ellipse (Fig. 3b), as described in Alexander (1985).

**Fire propagation**

The local model for fire ROS, together with the global model for fire shape, defines the physical behaviour of the fire. As input parameters are usually space-dependent owing to terrain topography and non-uniform fuels distribution, not to mention time dependence due to changing atmospheric conditions, simulations must be carried out on a step-by-step basis in order to account for the abovementioned spatial and temporal variations. In terms of implementation, the fire time evolution may be regarded either as vector- or as raster-based. In the former approach, the application of Huygens’ principle (Richards 1990) leads to the definition of the new fire front as the envelope of small ellipses computed locally from the previous fire front. For the raster approach, the one adopted in the present work, the calculation area is divided into rectangular- or other-shaped cells, where conditions are assumed as locally uniform. Following Dijkstra’s dynamic programming algorithm (Kourtz and O’Regan 1971), for each cell along the fire front, a contagion process is computed by assigning ignition instants to a predefined set of neighbour cells. Examination of all potential ignition candidates allows the identification of the next ignited cell – the corresponding ignition instant makes the solution advance in time. Thus, the time progression is not constant, depending on the ignition instant of each cell.

**Interaction between wind and fire**

Wind and fire calculations are carried out with different meshes. Each computational cell for wind calculation (wind cell) is defined by joining together several finer cells (fire cells) where fire propagation is calculated. Owing to the different resolution of wind and fire meshes, the fire front may occupy a fraction of a (coarse) wind cell, as exemplified in Fig. 4. The total heat release rate per unit volume $S_F$ at the cells adjacent to the ground, to be included in the source term $S_T$ in the energy conservation Eqn 4, is computed based on the reaction intensity

$$S_T = \frac{\sum_{n=1}^{nt} I_{rn} A_n}{v} \quad (30)$$

where $v$ is the coarse cell volume, $A_n$ is the surface area of each of the total $nt$ fine cells located inside the coarse cell and $I_{rn}$ represents the reaction intensity $I_r$ due to the fire in each fine cell $n$. Total heat released at each coarse cell is used to compute a new temperature distribution within the air, by solving Eqn 4, which affects buoyancy and turbulence production or destruction, through the source terms in Eqs 1, 9 and 10.

The calculation procedure consists in an alternation between the fire spread calculation and the wind calculation. After the fire area increases by a certain amount, the new heat release rate distribution at the ground is passed into the wind calculation module. After convergence of the fluid flow and energy equations, the updated wind field is used as input to continue the fire spread calculation.
Results

Herein, we present a sensitivity analysis of the effect of some numerical parameters on the calculation of the interaction between wind and fire. These include the criterion for wind update and the resolution adopted for the wind field calculation. In the companion paper (A. M. G. Lopes, M. G. Cruz and D. X. Viegas, unpubl. data), application of the models described to a laboratory fire and to a large-scale real fire are reported.

Fig. 2a shows visualisations of the wind field 10 m above ground and fire shape at different stages. From these figures, one can easily see the influence of topography on wind, as well as the air in-draft towards the fire region due to the convective plume from the fire. This is clearly visualised in Fig. 2d, where information on the air temperature field is included in a lateral view. In this representation, temperature is corrected for altitude, thus corresponding to potential temperature. Computed maximum temperatures in the fire region are around 240°C at vertical distances of 5 and 10 m above ground respectively.

Results depicted in Fig. 5 were obtained using a fire area growth of 150 ha as wind update criterion, i.e. wind field is

<table>
<thead>
<tr>
<th>Horizontal resolution (m)</th>
<th>Horizontal domain size (km)</th>
<th>Total number of nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1250</td>
<td>22.5 x 18.5</td>
<td>20 x 16 x 15 = 4800</td>
</tr>
<tr>
<td>750</td>
<td>22.5 x 18.5</td>
<td>31 x 26 x 15 = 12090</td>
</tr>
<tr>
<td>500</td>
<td>22.5 x 18.5</td>
<td>46 x 38 x 15 = 26220</td>
</tr>
<tr>
<td>250</td>
<td>22.5 x 18.5</td>
<td>91 x 76 x 15 = 103740</td>
</tr>
<tr>
<td>150</td>
<td>22.5 x 18.5</td>
<td>151 x 126 x 20 = 380520</td>
</tr>
<tr>
<td>100</td>
<td>8.6 x 9.4</td>
<td>87 x 95 x 20 = 165300</td>
</tr>
<tr>
<td>50</td>
<td>8.6 x 9.4</td>
<td>173 x 189 x 20 = 653940</td>
</tr>
</tbody>
</table>

Fig. 7. Influence of wind update criterion on: (a) calculation time; (b) fire size and shape 4 h after ignition. Gridlines in the graph are 1000 m apart. Red dot represents ignition location.
updated when the fire area increases 150 ha since the last update. A spatial resolution of 500 m for the wind mesh with 15 vertical levels was used. The influence of these parameters will now be investigated.

Study on dynamic interaction update frequency

For the first set of tests, to be presented next, wind is updated every time the fire area increases by a specific constant value since the last wind update. Fig. 6a shows the fire area evolution with time for different values of fire area increase. These results are compared with the ‘static’ case (no two-way coupling), for which the initial wind field, computed with no heat transfer effects, is considered at all stages of fire growth. The first noticeable characteristic is that fire spread is much slower when dynamic interaction is taken into account. This behaviour can be anticipated, because the buoyancy induced by the fire creates an inflow wind towards the fire region, which is opposed to fire growth direction, a clearly evident feature in Fig. 5b, c. The effect of wind update frequency influences fire development mainly in the early stages after fire start. As Fig. 6a shows, as soon as the wind is first updated, the change in the wind magnitude and direction causes the fire ROS to decrease. Further updates do not change the wind by as large an amount as the first update. This suggests that it is not that useful to update the wind based on relatively small amounts of fire area increase – a fractional increase would be a wiser criterion, mainly in terms of computational effort. In Fig. 6b, the results obtained using various values of fractional increase are shown, starting from the first wind update (Updt) at 50 ha. Thus, for example, for the case ‘Updt[50ha];2×’, the wind is updated for fire areas of 50, 100, 200, 400 and 800 ha, which leads to five wind calculation runs, instead of 20 runs for a constant area increase of 50 ha. As may be seen, even for the 3× case, fire growth does not differ markedly from the constant 50-ha situation in the simulated temporal region. This reflects, nevertheless, on computational time, because wind calculation is responsible for more than 99% of the computational effort, as may be appreciated from Fig. 7a. This emphasises the convenience of using a non-constant area increase as a wind update criterion. Fig. 7b, in turn, represents the fire shape 4 h after ignition, for several update modes, including the static (no interaction) case.

Study on wind calculation resolution

The choice of the spatial resolution adopted for the wind calculation mesh should be done with care. Higher mesh resolutions reduce the numerical discretisation errors of the equations. In the case of simulations for topography, the mesh size also dictates how correctly terrain features are described. It is a general rule that each geometrical feature should be meshed with at least 10 nodes. Nevertheless, owing to the fractal dimension of terrain characteristics, this criterion is virtually impossible to satisfy and, thus, truly mesh-independent results are not obtainable. In the present work, several spatial resolutions in the horizontal plane for the wind mesh were adopted, ranging from 50 m, which corresponds to the fire mesh resolution for fire spread calculation, up to 1250 m. For the coarser wind meshes, a larger domain was considered (depicted in Fig. 2a), in order to keep boundaries sufficiently far from fire in terms of calculation nodes. A constant number of 15 vertical levels was considered in all situations. Table 1 synthesises pertinent data for the different wind meshes. For these simulations, the dynamic interaction was updated starting at 50 ha of burned area, and using an increment factor of 2×.

Results concerning rate of fire area growth are presented in Fig. 8a. It may be seen that a considerable dependence on horizontal resolution exists, though, as expected and desirable, it is attenuated for finer wind meshes; in fact, it may be stated that results for 100- and 50-m resolution are reasonably similar. Fig. 8b shows that computation time increases in a strong non-linear manner with mesh resolution – in the interpretation of this graphic, one should keep in mind that the two finer-mesh data sets correspond to a smaller domain.

Results for fire shape 4 h after ignition are depicted in Fig. 9. Results for the three coarsest meshes show a marked dependency on mesh spacing. However, fire shapes for the two finer meshes (100 and 50 m) are rather similar. Visualisation of the wind flow, framed on the whole wind calculation domain for the 50-m mesh, is presented in Fig. 10. For better visualisation, a
sampling of one out of four vectors was performed in each direction, so the wind field is presented on a 200-m-spaced mesh. One may note the strong in-draft all around the fire perimeter. Fig. 11a, b shows a visualisation in a vertical plane perpendicular to the main wind direction (transect represented in Fig. 10), with colour contours for temperature and velocity magnitude. As the burning region assumes a doughnut-like shape, the central part, characterised by a lower heat release rate, is subject to air down-draft. The buoyant region on the flaming front, however, produces a rising plume, with a vertical wind component that is even stronger than the horizontal component, creating a large vortex surrounding the fire. In this figure, one may identify a typical dimension of ~500 m for the buoyant plume. Taking into consideration that a mesh size of 50 m was found to be appropriate for this case, this is consistent with a meshing requirement of 10 nodes for typical flow features.

Dependence of the results on the number of vertical levels is much weaker, as may be appreciated from Fig. 12, where all results were computed using a 100-m horizontal resolution, keeping the first vertical level 2 m above ground.

Conclusions

We propose a two-way coupling method for fire behaviour prediction, where the buoyancy effects caused by the fire heat release are fully simulated. The present paper describes the underlying models for wind field and fire spread calculation. The wind field calculation consists of a Navier–Stokes solver
applied to a boundary fitted mesh. For simplicity, the mesh is Cartesian in the horizontal plane and terrain-following in the vertical direction. This considerably simplifies the calculation procedure, leading to quite modest calculation times for a Navier–Stokes solver. Fire spread is computed with Rothermel’s model for surface fire, with a cell-based approach, where fire progression takes place through a contagion process. Fire shape is given by ellipse-type models. The wind field is computed on a mesh made by agglomerating cells from the fire calculation mesh. The reaction intensity given by the fire spread model provides the heat release rate to be used for computing buoyancy for the wind calculation.

We conducted a sensitivity analysis studying the influence of numerical parameters on the calculated wind field and fire shape for a simplified scenario. It is shown that the two-way coupling is most important in the early stages of fire development, but its importance decreases as the fire area gets larger. As the wind calculation represents the majority of the computational effort, it is advisable to rationalise the number of wind calculations. It is shown that updating the wind field at constant values for the fire area increase does not lead to significant differences on results when compared with an update based on an area fraction increase of $2 \times$ or even $3 \times$. Furthermore, calculation times are reduced to $1/5$ with the latter option. Studies of the influence of wind mesh resolution clearly indicate that the number of vertical

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**Fig. 11.** Vertical plane perpendicular to incident wind direction, passing though fire shape centre (transect depicted in Fig. 10): (a) temperature field; (b) velocity magnitude field.

**Fig. 12.** Dependence of fire shape on number of vertical levels, using a 100-m mesh.
calculation levels affects the solution less than the horizontal resolution. The classical concept of mesh-independent results on computational fluid dynamics is difficult to apply to wind over complex topography, owing to the fractal characteristics of terrain shape. Results indicate that, at least for the simulated cases, horizontal wind mesh spacing should not be larger than, typically, 50 to 100 m.

Conflicts of interest
The authors declare no conflicts of interest.

Table 2. Nomenclature

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{w}, a_{zh}$</td>
<td>coefficients in the general discretised equation</td>
</tr>
<tr>
<td>$A_{w}$</td>
<td>area of the coarse cell (m$^2$)</td>
</tr>
<tr>
<td>$A_{f}$</td>
<td>area of the fine cell (m$^2$)</td>
</tr>
<tr>
<td>$b$</td>
<td>generic source term</td>
</tr>
<tr>
<td>$c_{p}$</td>
<td>specific heat (J kg$^{-1}$ K$^{-1}$)</td>
</tr>
<tr>
<td>$B_{f}$</td>
<td>buoyancy force per unit volume, along direction $i$ (N m$^{-3}$)</td>
</tr>
<tr>
<td>$C_{p}, C_{1}, C_{2}, C_{3}$</td>
<td>constants for the turbulence model</td>
</tr>
<tr>
<td>$d$</td>
<td>distance between the first node and the ground (m)</td>
</tr>
<tr>
<td>$f$</td>
<td>ratio between the roughness parameter and the roughness height</td>
</tr>
<tr>
<td>$f_{d}$</td>
<td>initial reaction intensity cut-off after flame extinguishment</td>
</tr>
<tr>
<td>$g$</td>
<td>gravity acceleration (m s$^{-2}$)</td>
</tr>
<tr>
<td>$G$</td>
<td>production or destruction of turbulence due to thermal effects (kg m$^{-1}$ s$^{-3}$)</td>
</tr>
<tr>
<td>$h_{0}$</td>
<td>physical roughness height (m)</td>
</tr>
<tr>
<td>$h_{f}$</td>
<td>flame height (m)</td>
</tr>
<tr>
<td>$h_{v}$</td>
<td>vegetation height (m)</td>
</tr>
<tr>
<td>$I_{r}$</td>
<td>reaction intensity (J m$^{-1}$ s$^{-1}$)</td>
</tr>
<tr>
<td>$I_{zd}$</td>
<td>reaction intensity during decay phase (W m$^{-2}$)</td>
</tr>
<tr>
<td>$I_{z}$</td>
<td>reaction intensity in cell $n$ (W m$^{-2}$)</td>
</tr>
<tr>
<td>$J$</td>
<td>Jacobian (unitless)</td>
</tr>
<tr>
<td>$k$</td>
<td>turbulence kinetic energy (m$^2$ s$^{-2}$)</td>
</tr>
<tr>
<td>$L$</td>
<td>dissipation length scale (m)</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure (Pa)</td>
</tr>
<tr>
<td>$P_{s}$</td>
<td>production rate turbulence kinetic energy (kg m$^{-1}$ s$^{-3}$)</td>
</tr>
<tr>
<td>$P_{r}, P_{rt}$</td>
<td>Laminar and turbulent Prandtl number</td>
</tr>
<tr>
<td>$Q_{r}$</td>
<td>heat of pre-ignition (J kg$^{-1}$)</td>
</tr>
<tr>
<td>$R$</td>
<td>residual (unitless)</td>
</tr>
<tr>
<td>$R_{s}$</td>
<td>rate of spread with zero wind speed and zero terrain slope (m s$^{-1}$)</td>
</tr>
<tr>
<td>$R_{g}$</td>
<td>rate of spread (m s$^{-1}$)</td>
</tr>
<tr>
<td>$S_{t}$</td>
<td>energy transfer rate with the ground (J s$^{-1}$ m$^{-3}$)</td>
</tr>
<tr>
<td>$S_{p}$</td>
<td>heat release rate due to fire in each cell (J s$^{-1}$ m$^{-3}$)</td>
</tr>
<tr>
<td>$S_{t'}$</td>
<td>energy transfer rate with the ground (J s$^{-1}$ m$^{-3}$)</td>
</tr>
<tr>
<td>$t$</td>
<td>time (s)</td>
</tr>
<tr>
<td>$t_{r}$</td>
<td>relaxation time (s)</td>
</tr>
<tr>
<td>$T_{r}$</td>
<td>turbulence intensity (unitless)</td>
</tr>
<tr>
<td>$u, v, w$</td>
<td>velocity components along the $x, y, z$ coordinates (m s$^{-1}$)</td>
</tr>
<tr>
<td>$u_{f}$</td>
<td>generic velocity component (m s$^{-1}$)</td>
</tr>
<tr>
<td>$U_{f}$</td>
<td>generic contravariant velocity component (m s$^{-1}$)</td>
</tr>
<tr>
<td>$V$</td>
<td>wind velocity (m s$^{-1}$)</td>
</tr>
<tr>
<td>$V_{s}$</td>
<td>wind velocity at the top of the boundary layer (m s$^{-1}$)</td>
</tr>
<tr>
<td>$V_{s'}$</td>
<td>shear velocity (m s$^{-1}$)</td>
</tr>
</tbody>
</table>

(Continued)


Kourtz PH, O’Regan WG (1971) A model for a small forest fire ... to simulate burned and burning areas for use in a detection model. Forest Science 17(2), 163–169.


